Effect of Positive Ions on Drift Field

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Introduction

A minimum ionizing particle generates 50,000 ionization electrons and an equal number of positive ions per cm of path length in liquid argon. The electrons move, in an electric field of 500 V/cm, at a speed of $\mathrm{v}_{\mathrm{drift}} = 1500$m/s towards the readout wires (anode) where they produce signals on the sensing planes. The positive ion mobility is a factor of 3 E5 less [1] and the positive ions move with a speed of about 0.5 cm/s towards the cathode. The question has been raised as to what effect the positive ions generated by cosmic rays (in a surface detector) will have on the drift field.

Derivation of Effect

I follow exactly the method of reference 2 [2] changing only the names of some variables. The continuity equation is written as ..

$$\frac{\partial \rho}{\partial t} = I - \nabla . J \tag{1}$$

where ρ is the charge density in the liquid, I is the rate of formation of positive ion charge, and J is the current of positive ions (towards positive z). Reference 2 uses J where I have I. In the steady state

$$\frac{\partial \rho}{\partial t} = 0 \tag{2}$$

Substituting $J = \mu E \rho$ and assuming drift in the z direction only, we get

$$\frac{\partial(\mu E \rho)}{\partial z} = I \tag{3}$$

whence

$$\mu E \rho = I z + c \tag{4}$$

where $c = 0$ since the positive ion density is nil at the anode.
One can substitute for ρ from Maxwell's equation viz:

$$\nabla . E = \frac{\partial E}{\partial z} = \frac{\rho}{\epsilon} \tag{5}$$

to get the equation for the electric field in terms of the rate of creation of positive ion charge (we haven't imposed all the boundary conditions so there is an unknown constant at this stage)

$$\mu\epsilon E\frac{\partial E}{\partial z} = Iz \tag{6}$$

$$E^2 = \frac{I}{\mu\epsilon}z^2 + C \tag{7}$$

Since the field cannot reverse direction between the anode and the cathode, the minimum value of C is 0, so we can write:

$$E = \sqrt{C + \frac{I}{\mu\epsilon}z^2}. \tag{8}$$

Setting $C = 0$ shows a limit to the validity of this approach. If the total gap is D and the applied voltage is V, then

$$\int_0^D Edz = V = \frac{1}{2}\sqrt{\frac{I}{\mu\epsilon}}D^2 \tag{9}$$

which implies for any $C > 0$

$$\frac{D^2}{V}\sqrt{\frac{I}{\mu\epsilon}} \equiv \alpha < 2 \tag{10}$$

In our case, $\mu = 1.6 \times 10^{-7} m^2/V - s$, $\epsilon = 14 \times 10^{-12}$ Farads/m, (the relative permittivity of liquid argon is 1.6), I is the product of the rate of cosmics rays passing through a cubic meter per second and the amount of charge liberated by each ray in their ~ 1 meter passage $= 200 \times 5 \times 10^6 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-10}$ Coulombs/sec, $D = 3\ m$ and $V = 1.5 \times 10^5$ volts. This gives $\alpha = 0.51$ well within the model.

The authors of reference 2 [2] do something nice which is to express the field in terms of α and the field in the case of no positive ions, viz

$$E = \frac{V}{D}\sqrt{\hat{c}^2 + \alpha^2\frac{z^2}{D^2}}. \tag{11}$$

where $\frac{V}{D}$ is the field, E_0, if there were no ions.

The parameter $\hat{c}$ gives the field at $z = 0$ as a fraction of E_0 To derive the value of $\hat{c}$ one takes $\int_0^D Edz$ and equates the answer to V. The integral of the form $\int_0^D \sqrt{a^2 + z^2}dz$ can be found in standard tables; the answer involves square-roots and logarithms and I don't know an analytic solution for $\hat{c}$. Reference 2 has a convenient plot which I have reproduced for fun, shown here as figure 1. It gives $\hat{c}$ (the fraction of E_0 at the anode) and $\sqrt{\hat{c}^2 + \alpha^2}$, the multiplier for E_0

at the cathode as a function of α. The field has to integrate to V and so varies by $\sim \pm(1-\hat{c})$ from cathode to anode.

As a sanity check, let's consider ICARUS T300 on the surface with a 1.5 m drift, $\alpha = 0.25$. The fractional effect on the electron drift velocity is about 1/2 the fractional change in the field at 500 V/cm. The field across the gap varies as $1 + 0.5 \times \alpha^2 \frac{z^2}{D^2}$ (where I've set $\hat{c} = 1$) so the drift-velocity will vary as $1 + 0.25 \times \alpha^2 \frac{z^2}{D^2}$; this gives a total variation in drift-velocity of 1.6%. (I know two wrongs do not make a right, but this is about the variation in drift-velocity from temperature variations of $\pm 0.5K$ [3]).

It would be nice to estimate the effect of the variation in the field on a track for the drift-distance if one does not compensate for the effect of positive ions in the drift-field and assumes a constant drift-velocity. Figure 2 shows the 'error' on the transverse distance for a 3 meter drift ($\alpha = 0.5$) as a function of z taking two simple cases for the drift-velocity.

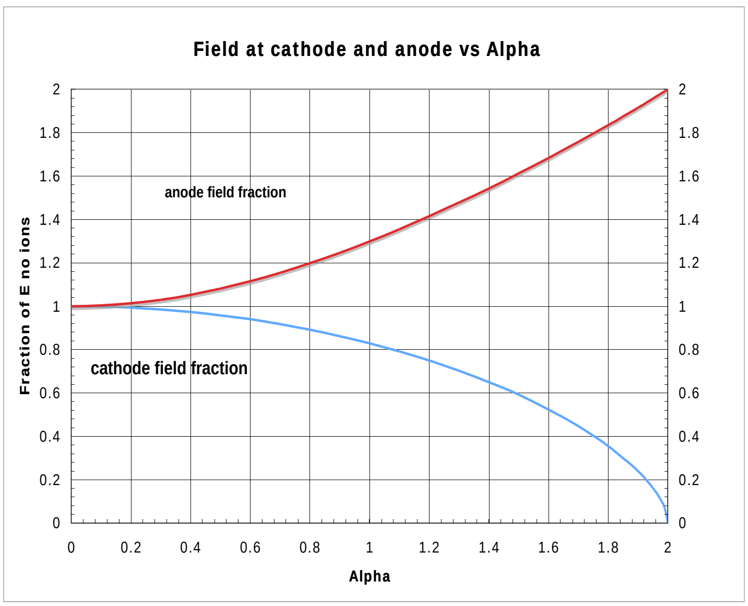

Figure 1: Variation of anode and cathode fields with α

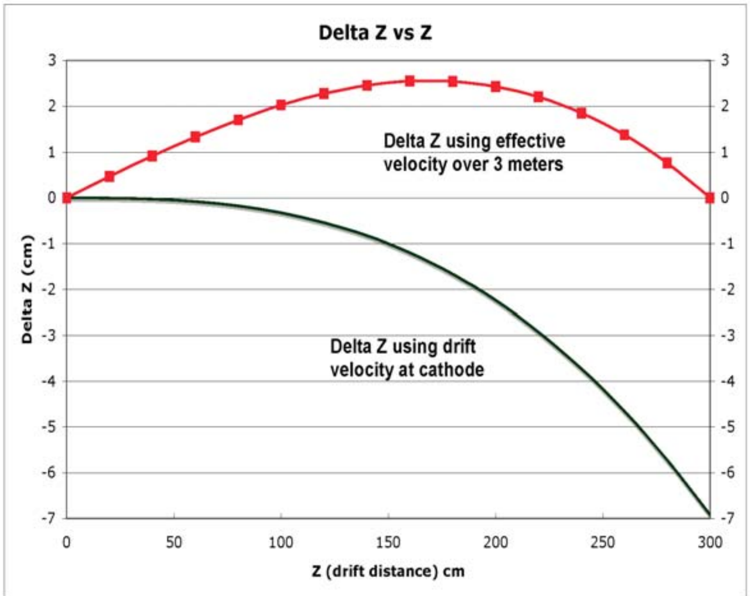

Figure 2: Transverse error versus distance from anode

References

[1] Data from the Atlas TDR in http://lartpc-docdb.fnal.gov/cgi-bin/ShowDocument?docid=206

[2] S. Palestini *et al.*, **NIM A421** (1999) 75-89 and http://lartpc-docdb.fnal.gov/cgi-bin/ShowDocument?docid=160

[3] W. Walkowiak, **NIM A449** (2000) 288-294 and http://lartpc-docdb.fnal.gov/cgi-bin/ShowDocument?docid=206